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For $i = 1, \ldots, n$, let x_i denote the subjective probability that the ith challenged call will be reversed. Let y_i be a random variable that equals 1 if the ith challenged call is reversed, and 0 otherwise. If x_i is the true probability that $y_i = 1$, then

$$\mathcal{E}(y_i|x_i) = x_i \tag{1}$$

and

$$\text{var}(y_i|x_i) = x_i(1 - x_i). \tag{2}$$

We can estimate the parameters of the regression

$$\mathcal{E}(y_i|x_i) = \beta_1 + \beta_2 x_i + \beta_3 x_i^2 \tag{3}$$

by Generalized Least Squares (GLS). If the x_i are the true reversal probabilities, then β_1 and β_3 equal 0 and β_2 equals 1. If the estimates differ from those values by more than reasonable sampling error, it will suggest that the subjective probabilities are biased.

Let y be the column vector of length n with ith element y_i, and let X be the $n \times 3$ matrix with ith row

$$\begin{bmatrix} 1 & x_i & x_i^2 \end{bmatrix}. \tag{4}$$

Let D be the $n \times n$ diagonal matrix with ith diagonal element

$$d_{ii} = \frac{1}{x_i(1 - x_i)}, \tag{5}$$

and define vectors

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \quad \text{and} \quad \beta_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \tag{6}$$

Then under the null hypothesis that the x_i are the true reversal probabilities, and conditional on the x_i, the GLS estimate

$$\hat{\beta} = (X^t D X)^{-1} X^t D y \tag{7}$$

has mean β_0 and covariance matrix $\Omega = (X^t D X)^{-1}$. If n is large enough, $\hat{\beta}$ will be approximately multivariate normal, and the test statistic

$$s = (\hat{\beta} - \beta_0)^t \Omega^{-1} (\hat{\beta} - \beta_0) \tag{8}$$

will have approximately a χ^2 distribution with 3 degrees of freedom. Large values of s cast doubt on the null hypothesis. However, with the actual data, the observed test statistic is $s = 1.2$, which is only the 25th percentile of the χ^2_3-distribution. (We get the same percentile if, instead of assuming that s is χ^2, we determine its distribution by simulation.) Therefore, the deviation of $\hat{\beta}$ from β_0 is actually somewhat *smaller* than one would expect from chance alone, if the null hypothesis is true. So, the test furnishes no evidence that the subjective probabilities are biased.